The singular perturbations for the abstract semi-linear evolution equations in Hilbert spaces

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Let H and V are two real Hilbert spaces, such that V is continuously embedded in H. In the space H we consider the following Cauchy problem:

$$\begin{cases} \varepsilon u_\varepsilon''(t) + u_\varepsilon'(t) + Au_\varepsilon(t) + B\big(u_\varepsilon(t)\big) = f_\varepsilon(t), \quad t \in (0,T), \\ u_\varepsilon(0) = u_{0\varepsilon}, \quad u_\varepsilon'(0) = u_{1\varepsilon}, \end{cases} \qquad (P_\varepsilon)$$

where $A : V \subset H \to H$ is a linear self-adjoint operator and B is nonlinear $A^{1/2}$ lipschitzian or monotone operator, $u_{0\varepsilon}, u_{1\varepsilon} \in H, f_\varepsilon : [0,T] \to H$ and ε is a small parameter. We will study the behaviour of solutions to the problem (P_ε), as $\varepsilon \to 0$, relativ to the correspondig solutions to the unperturbed problem (P_0):

$$\begin{cases} v'(t) + Av(t) + B\big(v(t)\big) = f(t), \quad t \in (0,T), \\ v(0) = u_0. \end{cases} \qquad (P_0).$$

If $||u_{0\,\varepsilon} - u_0||_V \to 0$, $||u_{1\,\varepsilon} - u_1||_H \to 0$, $||f_\varepsilon - f||_{W^{1,2}(0,T;H)} \to 0$, using the relationship between solutions of the systems (P_ε) and (P_0) and a priori estimates of solutions to the system (P_ε) we prove that

$$u_\varepsilon \to v \quad \text{in} \quad C([0,T];H) \cap L^\infty(0,T;V), \quad \text{as} \quad \varepsilon \to 0.$$

This means that the perturbations of the system (P_0) by the system (P_ε) are regular in the indicated norms. In the same time, we show that

$$u_\varepsilon' - v' - \alpha_\varepsilon\, e^{-t/\varepsilon} \to 0 \quad \text{in} \quad C([0,T];H) \cap L^\infty(0,T;V) \quad \text{as} \quad \varepsilon \to 0, \qquad (1)$$

where $\alpha_\varepsilon = f_\varepsilon(0) - u_{1\varepsilon} - Au_{0\varepsilon} - B(u_{0\varepsilon})$. It means that the derivatives of solutions to the problem (P_ε) do not converge to derivatives of the corresponding solutions of problem (P_0), when $\varepsilon \to 0$. The relation (1) shows that the derivative u' has a singular behaviour relative to $\varepsilon \to 0$ in the neighborhood of $t = 0$. This singular behaviour is determined by the function $\alpha_\varepsilon\, e^{-t/\varepsilon}$, which is *the boundary layer function* and the neighborhood of $t = 0$ is *the boundary layer* for u'.

The mathematical model (P_ε) governs various physical processes, which are described by the Klein-Gordon equation, the Sine-Gordon equation, the plate equation, the Cahn-Hilliard equation and others equations. Therefore, we apply these results to the indicated equations.

We also study the behavior of solutions to the following system:

$$\begin{cases} \varepsilon u_{\varepsilon\delta}''(t) + \delta\, u_{\varepsilon\delta}'(t) + Au_{\varepsilon\delta}(t) + B\big(u_{\varepsilon\delta}(t)\big) = f_\varepsilon(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_{0\varepsilon}, \quad u_{\varepsilon\delta}'(0) = u_{1\varepsilon}, \end{cases} \qquad (P_{\varepsilon\delta})$$

when $\varepsilon \to 0$ and $\delta \to 0$ simultaneously.